ACCELERATION OF CONDUCTORS IN THE PULSED MAGNETIC FIELD OF A MASSIVE CYLINDRICAL MULTITURN INDUCTOR

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The force of a pulsed magnetic field on conductors is used in the induction-dynamic drive of switching devices, in the magnetic pulse processing of metals, and in the high-speed induction projection of conductors. In investigating electromagnetic processes the penetration of the electromagnetic field (EMF) into conductors is generally treated in the quasistationary approximation. In [1] the magnetic vector potential (MVP) A was chosen as the fundamental electromagnetic quantity, and the appropriate equations were written down. In [2, 3] two-dimensional quasistationary EMF were calculated by the integral equations method (IEM), and in [4, 5] by the finite difference method (FDM). In [6] an equivalent circuit was used to investigate a "cylindrical" pulsed-dynamic system (PDS); the inductor and conductor were replaced by hollow cylinders of thickness equal to the equivalent depth of penetration of the EMF for a frequency corresponding to the discharge of a capacitor bank (CB) through a "dead" inductor. In [7] the IEM was used to investigate a PDS consisting of a multiturn inductor in the form of a solenoid and an armature in the form of a massive cylinder, neglecting the penetration of the EMF into the inductor, which is the case in systems of the type considered, particularly for the acceleration of bodies of small mass, when it is necessary to decrease the inductance of the inductor in order to achieve optimum transformation of the energy of the CB into energy of the accelerated body. In this case it is necessary to employ systems whose inductors have a small number of massive turns. Such inductors are preferable also from the point of view of Joule heating. The IEM can be used to investgate PDS when account is taken of the penetration of the EMF into the inductor [7], but in this case a region of primary currents is also developed in parts with uniform properties, which leads to an appreciable increase in the number of unknowns in the system of difference equations. The order of the system of differential equations obtained with the IEM is lower than that obtained with the FDM, but the matrix of the coefficients is completely filled, while a band matrix is obtained with the FDM. In certain cases the FDM is more advantageous because of the existence of programs which require less machine time and storage [5]. However, in using the FDM for systems with multiturn inductors it is necessary to calculate not only the spatial distribution of the MVP, but also to determine the time dependence of the voltages across the inductor turns.

In the present article we present a computational scheme and the results of a mathematical study of the acceleration of conductors in the pulsed magnetic field of a massive cylindrical multiturn inductor through which a CB is discharged (Fig. 1), taking account of the penetration of the field into the inductor and conductor. The accelerated conductor is a solid cylinder of mass m coaxial with the inductor. In general this system is not axisymmetric, and its investigation requires the computation of a three-dimensional EMF. However, the larger the diameter of the inductor and the smaller the width and pitch of a turn, the more properly can a multiturn inductor be considered axisymmetric, and a two-dimensional EMF calculated in the (r, z) plane. There are PDS with an inductor consisting of series connected rings. Since in the present article we treat the problem in the axisymmetric approximation, the MVP has only one nonzero component A = {0, A, 0}.

As basic quantities we choose the following:

$$t_b=\sqrt[3]{\frac{\mu_0}{\gamma_b}(2\pi C)^2},\quad x_b=\sqrt[3]{\frac{2\pi C}{\mu_0\gamma_b^2}},\quad E_b=\frac{CU_0}{\gamma_b x_b^2 t_b},\quad v_b=\frac{x_b}{t_b},\quad A_b=\frac{t_b U_0}{2\pi x_b},\quad i_b=\frac{CU_0}{t_b},\quad m_b=\frac{CU_0^2}{v_b^2},$$

where μ_0 is the permeability of vacuum, C is the capacitance of the CB, γ_b is the electrical conductivity of the inductor material, and U_0 is the initial voltage across the CB.

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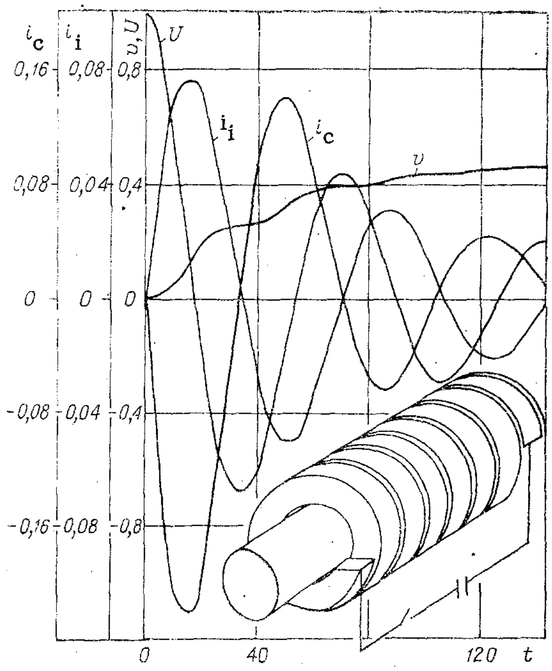

Fig. 1

Supplementing the electromagnetic equations in [1] by equations describing the mechanical processes involved in the displacement of conductors, as was done in [5], and the equation for the CB circuit, we obtain the following system of equations for the relative quantities:

$$\oint_L \operatorname{rot} \mathbf{A} d\mathbf{l} = \int_S \gamma E ds;$$

$$E = \begin{cases} -\dfrac{\partial A}{\partial t} + \dfrac{U_k(t)}{r} & \text{in the k-th turn, } k = 1, \ldots, N, \\ -\dfrac{\partial A}{\partial t} - v\dfrac{\partial A}{\partial z} & \text{in the conductor;} \end{cases}$$

$$\sum_{k=1}^{N} U_k(t) = 1 - \int_0^t i dt - L_0 \frac{di}{dt} - R_0 i;$$

$$i_k = \int\int_{S_k} \gamma E ds;$$

$$i_k = i, \quad k = 1, \ldots, N;$$

$$m\frac{dv}{dt} = \int\int_{S_c} \gamma_c \left(-\frac{\partial A}{\partial t} - v\frac{\partial A}{\partial z}\right)\frac{\partial A}{\partial z} r ds;$$

$$\frac{dx}{dt} = v$$

with the initial conditions $A(0, r, z) = 0$, $v(0) = 0$, $x(0) = x_0$, and the boundary conditions $A(t, 0, z) = 0$, $A_{\substack{r\to\infty \\ z\to\infty}} = 0$. Solving Eqs. (1)-(7), we obtain the distribution of the MVP in the (r, z) plane and in time. In addition, we determine the time dependence of the voltages U_1, ..., U_N across the turns of the inductor, the velocity v of the conductor, and its position x. Using these relations, we find from (2) the current density distribution, and from (4) the currents in the turns of the inductor and in the conductor. Since we do not consider Joule heating of the conductors in the present article, we assume that the electrical conductivity γ_b of the inductor and $\gamma_c\gamma_b$ of the conductor are constant during the acceleration process. This assumption is valid when the magnetic induction is smaller than a critical value h_c [8], i.e., so long as thermal effects do not begin to have an appreciable effect on the penetration of the magnetic field into conductors. Since the problem under consideration is axisymmetric, we solve it in the half-plane $\{(r, z), r \geqslant 0\}$ in which artificial boundaries $z \leqslant z \leqslant z_k$ and $0 \leqslant r \leqslant R_M$ are chosen at distances from the conductors such that the boundary conditions at infinity are adequately satisfied on them. Thus, we assume that along the lines $z = z_0$, $z = z_k$, and $r = R_M$ the MVP is zero. In the calculations the boundaries were chosen at a distance of five characteristic lengths from the conductors, with a characteristic length taken as the maximum geometrical dimension of the system. Calculations showed that the error of this assumption was <1%. Thus, the exterior problem is reduced to an interior problem.

In the domain considered we lay out the following net in the (r, z) plane:

$$\Omega = \{(z_j,\ r_i),\ z_1 = h_1,\ z_{j+1} = z_j + h_{j+1},\ j = 1, \ldots, K-1;$$
$$r_1 = h_1,\ r_{i+1} = r_i + h_{i+1},\ i = 1, \ldots, M-1\}.$$

We take $\hbar_i = 0.5(h_i + h_{i+1})$, $\hbar_j = 0.5(h_j + h_{j+1})$, and then, combining Eqs. (1) and (2), and writing them in difference form, we obtain the following equation for the MVP:

$$\gamma \frac{A_{i,j}^n - A_{i,j}^{n-1}}{\Delta t} = \Lambda_1 A^n + \Lambda_2 A^n + f_{i,j}^n,$$

where

$$\Lambda_1 A^n = \frac{1}{\hbar_i}\left[-\frac{r_i A_{i,j}^n - r_{i-1}A_{i-1,j}^n}{r_{i-\frac{1}{2}} h_i} + \frac{r_{i+1}A_{i+1,j}^n - r_i A_{i,j}^n}{r_{i+\frac{1}{2}} h_{i+1}}\right],$$

$$\Lambda_2 A^n = \frac{1}{\hbar_j}\left[\frac{A_{i,j+1}^n - A_{i,j}^n}{h_{j+1}} - \frac{A_{i,j}^n - A_{i,j-1}^n}{h_j}\right],$$

$$f_{i,j}^n = \begin{cases} U_k^n / r_i & \text{in the k-th turn } (k = 1, \ldots, N), \\ -v\left(\dfrac{\partial A}{\partial z}\right)_{i,j} & \text{in a conductor,} \\ 0 & \text{outside conductors,} \end{cases}$$

$$\left(\frac{\partial A}{\partial z}\right)_{i,j} = \frac{1}{2\hbar_j}\left[\frac{h_j\left(A_{i,j+1}^n - A_{i,j}^n\right)}{h_{j+1}} + \frac{h_{j+1}\left(A_{i,j}^n - A_{i,j-1}^n\right)}{h_j}\right].$$

We write Eqs. (3), (4), (6), and (7) in finite difference form:

$$\sum_{k=1}^{N} U_k^n = 1 - \sum_{l=1}^{n} \frac{i^l + i^{l-1}}{2}\Delta t,$$

$$i_k^n = \sum_{i,j \in S_k}\left(\frac{U_k^n}{r_i} - \frac{A_{i,j}^n - A_{i,j}^{n-1}}{\Delta t}\right)\hbar_i \hbar_j, \quad k = 1, \ldots, N,$$

$$m\frac{v^{n+1} - v^n}{\Delta t} = -\sum_{i,j \in S_c} \gamma_c r_i \left[\frac{A_{i,j}^n - A_{i,j}^{n-1}}{\Delta t} + v^n\left(\frac{\partial A}{\partial z}\right)_{i,j}\right]\left(\frac{\partial A}{\partial z}\right)_{i,j}\hbar_i \hbar_j,$$

$$(x^{n+1} - x^n)/\Delta t = (v^{n+1} + v^n)/2.$$

The equation for the MVP is parabolic in conductors and elliptic outside them. In the region with $\gamma = 0$ we introduce the parameter $\gamma_v \ll \gamma_b$. It was shown in [5] that this method permits the use of the method of alternating directions, and the solution for $\gamma_v/\gamma_b \leqslant 10^{-2}$ is almost independent of γ_v. Since the results of our calculations were strongly dependent on γ_v, we used an iterative method with a stabilizing correction [9]. In this method the transition from step n − 1 to step n is accomplished by an iterative process

$$(A^{\nu-1/2} - A^{n-1})/\Delta t^{n-1} = \Lambda_1 A^{\nu-1/2} + \Lambda_2 A^{\nu-1} + f^{\nu-1},\ (A^{\nu} - A^{\nu-1/2})/\Delta t^{n-1} = \Lambda_2(A - A^{\nu-1}).$$

The iteration is stopped when Ampere's law

$$\oint_L \operatorname{rot} A dl = I_S$$

is satisfied with a specified error, where S is the area bounded by contour L, and I_S is the total current flowing through surface S. As contour L we chose L_1 — the z axis closing at infinity — and L_2 — the contour encompassing all the turns of the inductor.

Calculations in [5] were performed on a moving net. The net was moved in such a way that the boundary of the conductor was half way between the nodes of the net, which permitted

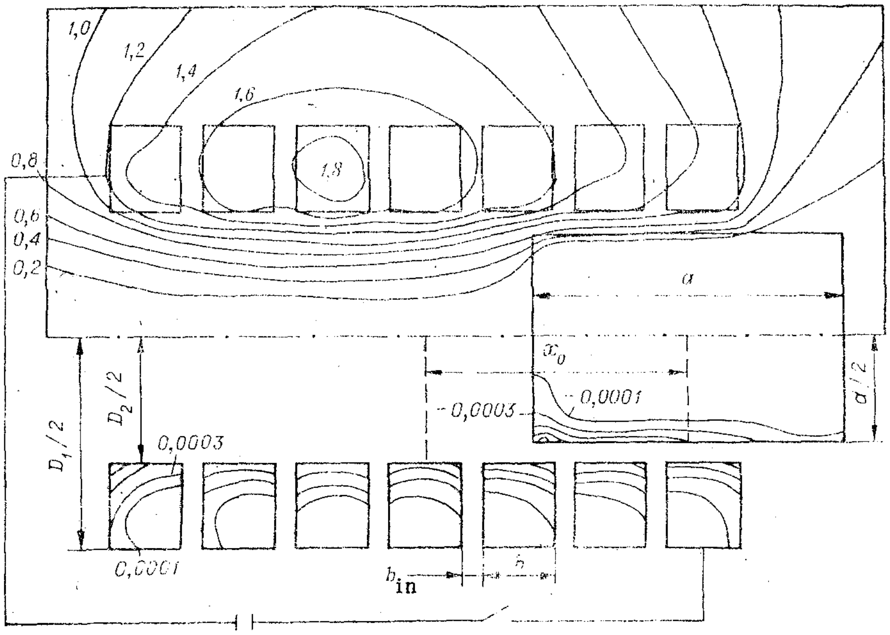

Fig. 2

the use of the scheme of through calculation. Such a net cannot always be chosen for the system we are considering, and therefore we performed the calculations on a stationary net, taking account of irregular nodes on the boundary of the conductor.

In order to test the efficiency of our difference schemes for the problem of the acceleration of conductors by a massive multiturn inductor, we calculated the discharge of a CB through a conductor of N = 7 turns of thickness equal to the equivalent depth of penetration of the magnetic field. A comparison of the calculated values of the current in the inductor with those obtained analytically for an inductor with a uniform current density distribution showed that the error of the calculation was <1%.

A series of calculations of the acceleration of conductors by the method described above completely confirmed the qualitative conclusions in [6]. Good agreement was obtained in the range of optimum parameters for the systems studied. However, in the range of variation of system parameters considered, the value of the velocity of an accelerated conductor calculated with an equivalent circuit exceeds the value obtained by the proposed method by 20% for $R_0 = 0$ and $L_0 = 0$, and by 10% for the actual values of R_0 and L_0. Obviously a systematic error can be accounted for by the approximate method of taking account of the current distribution over the surface of the conductors, which leads to an overestimate of the mutual inductance in the calculation by an equivalent circuit. On the basis of the simplicity of the model and the small expenditures of machine time, which is particularly important in solving design problems, we conclude that "cylindrical" PDS can be studied with sufficient accuracy ($\sim\pm5\%$) by using equivalent circuits if the values of the velocity obtained in the calculation of actual systems ($L_0 \neq 0$, $R_0 \neq 0$) are multiplied by 0.9. The proposed procedure for investigating electromagnetic processes should be used to determine the time dependence of the voltages across the turns of the inductor, the distribution of local characteristics over the cross section of a conductor (e.g., the current density, the radial and axial stresses), and for refining the results obtained in calculations with an equivalent circuit.

We present below the results of a calculation for a version with the following parameters (Fig. 2): $x_0 = 0.025$ m, $D_1 = 0.04$ m, $D_2 = 0.024$ m, $d = 0.02$ m, $a = 0.03$ m, $b = 0.07$ m, $b_{in} = 0.002$ m, N = 7, C = 0.02 F, $U_0 = 2$ kV, and m = 0.03 kg, a brass inductor ($\gamma = 14.7 \cdot 10^6$ 1/Ωm, and a chromium copper conductor ($\gamma = 27.2 \cdot 10^6$ 1/Ω·m). The graphs in Fig. 1 show the time dependence of the inductor current i_i, the conductor current i_c, the voltage U across the inductor, and the velocity v of the conductor. In this case the conductor is accelerated during almost the whole process of discharge of the CB through the system. The conductor experiences practically no negative accelerations, but during parts of the motion it moves with a constant velocity. The curves in Fig. 3 show the time dependences of the voltages across the turns in relative units. The voltages across the turns depend not only on their positions relative to the plane of symmetry of the inductor, but also on the posi-

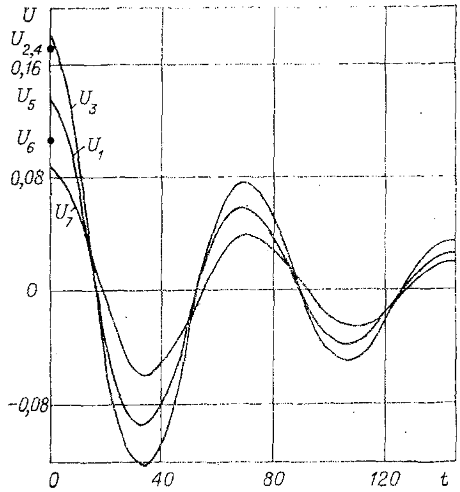

Fig. 3

tion of the conductor relative to the inductor. For convenience Fig. 3 shows curves for three turns only. The qualitative pattern for the remaining turns is similar. The values of the voltages at $t = 0$ for the second, fourth, and sixth turns are plotted along the axis of ordinates. The values of the voltages across the second, third, and fourth turns are negligibly different from one another.

The numerical values of the magnetic field appear alongside the field lines shown in Fig. 2. The figure also shows the current density distribution over the cross sections of the conductor and the turns of the inductor at the instant of the first current maximum ($t/t_b = 17$). It is clear from Fig. 2 that the center of the magnetic field lines is displaced to the third turn due to the presence of the conductor. This is accounted for by the fact that the voltages across the first and fifth turns are the same, and differ slightly from the voltages across the second, third, and fourth turns. The displacement of the center of the EMF lines in a similar way affects the current density distribution over the cross sections of the turns. In addition, it follows from Fig. 2 that the current is distributed practically over the inside of the inductor only, due to a combination of the proximity effect and the "bobbin" effect. Therefore, in contrast with "two-dimensional" systems, in this case the current density redistribution only over the "working" surface of the inductor is more pronounced. Calculations for various inductor thicknesses for a constant accelerated mass showed that for inductors thicker than two equivalent skin layers (with the discharge frequency determined by assuming the discharge through a "dead" inductor) the final velocity of the conductor was practically unchanged. For a decrease in the thickness of the inductor to less than two equivalent skin layers, the velocity of the conductor is decreased. This is caused by the decrease of the current in the inductor, which in turn occurs because of an increase in its resistance. For an inductor thickness equal to an equivalent skin layer, the velocity of the conductor is ∿75% of the limiting value.

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CRACK GROWTH IN A SATURATED POROUS MEDIUM DUE TO PASSAGE OF A CURRENT PULSE

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A solution has been obtained [1] for the growth of a crack in a continuous medium in response to the thermoelastic stresses produced by passing a current perpendicular to the crack. Here we consider a model describing the action of a current pulse on a saturated porous medium when the current flows through a crack filled with liquid of high electrical conductivity. It is assumed that the medium has a skeleton of low electrical conductivity and is penetrated by capillaries filled with an electrically conducting liquid. Then the effective conductivity σ_0 is determined by the microcapillary conductivity. We consider the case where the direct current is passed through two coaxial elliptical cracks filled with liquid having a high conductivity σ_1. The crack opening is characterized by the parameter $\beta = c/l$, where c is crack width and l is length. If the crack is very much open ($\beta \gg \sigma_0/\sigma_1$), the current supplied to the center of the crack will emerge from the ends, and near the ends the current density will be maximal, as will the corresponding ohmic losses. The heating in the pores increases the pore pressure and may cause the medium to fail at the crack vertex. Here we use methods from the theory of complex variable functions to solve the two-dimensional problem on the current-density distribution around a crack, and the Biot theory [2] is used to discuss the consolidation of ground and to estimate the parameters of the electrical pulse that disrupts the medium.

1. Current-Density Distribution. We consider the current-density distribution when the current flows through two elliptical cracks, with the source and antisource at the centers of these. We assume that the current is supplied through parallel infinitely long electrodes whose transverse dimension is much less than the crack width. The conductivity of the electrodes is much larger than that of the liquid within the cracks. This enables one to restrict consideration of the planar two-dimensional case. Figure 1 shows the track geometry. The potential distribution in a plane perpendicular to the electrodes satisfies the Laplace equation [3]

$$\operatorname{div}(\sigma_\nu \nabla \varphi_\nu) = -(I/2\pi)[\delta(z - z_1) - \delta(z - z_2)], \tag{1.1}$$

where $\nu = 1$ or 0. Here the potential φ_ν with subscript $\nu = 0$ corresponds to the region outside a crack, while that with subscript $\nu = 1$ corresponds to the region within the crack, $z = x + iy$ is the complex variable, x and y are Cartesian coordinates, z_1 and z_2 are the coordinates of the centers of the first and second ellipses correspondingly, I is the current injected into the crack per unit electrolength, and $\delta(z)$ is a Dirac function. The conditions for continuity of the potential and the normal component of the current density should be satisfied on the crack. We introduce the complex potential $F_\nu = \sigma_\nu \varphi_\nu$ and write these conditions in the form

$$\operatorname{Re}F_0 = \alpha \operatorname{Re}F_1; \tag{1.2}$$

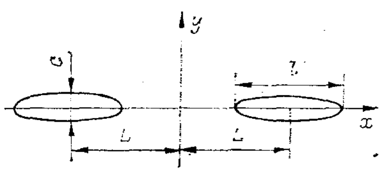

Fig. 1

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